

Experimental Evaluation of Distributed Node Coloring Algorithms for Wireless Networks

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Abstract

In this paper we evaluate distributed node coloring algorithms for wireless networks using the network simulator Sinalgo [1]. All considered algorithms operate in the realistic signal-to-interference-and-noise-ratio (SINR) model of interference. We evaluate two recent coloring algorithms, RAND4DCOLOR and COLORREDUCTION (in the following COLORRED), proposed by Fuchs and Prutkin in [2], the MW-Coloring algorithm introduced by Moscibroda and Wattenhofer [3] and transferred to the SINR model by Derbel and Talbi [4], and a variant of the coloring algorithm of Yu et al. [5]. We additionally consider several practical improvements to the algorithms and evaluate their performance in both static and dynamic scenarios.

Our experiments show that RAND4DCOLOR is very fast, computing a valid (4Δ) -coloring in less than one third of the time slots required for local broadcasting, where Δ is the maximum node degree in the network. Regarding other $\mathcal{O}(\Delta)$ -coloring algorithms RAND4DCOLOR is at least 4 to 5 times faster. Additionally, the algorithm is robust even in networks with mobile nodes and an additional listening phase at the start of the algorithm makes RAND4DCOLOR robust against the late wake-up of large parts of the network.

Regarding $(\Delta+1)$ -coloring algorithms, we observe that COLORRED is significantly faster than the considered variant of the Yu et al. coloring algorithm, which is the only other $(\Delta+1)$ -coloring algorithm for the SINR model. Further improvement can be made with an error-correcting variant that increases the runtime by allowing some uncertainty in the communication and afterwards correcting the introduced conflicts.

1 Introduction

Distributed node coloring is the underlying problem for many fundamental issues related to establishing efficient communication in wireless ad hoc and sensor networks. We can, for example, reduce the problems of establishing a time-, code-, or frequency-division-multiple-access (TDMA, CDMA, FDMA) schedule to a node coloring problem [6]. In this work we study and experimentally evaluate distributed node coloring algorithms that were designed for the realistic signal-to-interference-and-noise-ratio (SINR) model of interference. This model is widely used for decades in the electrical engineering community and was adopted by the algorithmic community after a seminal work by Gupta and Kumar [7]. In contrast to graph-based models, the SINR model reflects both the local and the global nature of wireless transmissions. However, to analytically prove guarantees on the runtime and show an algorithms correctness becomes relatively complex. Thus, over the past years techniques were developed to tackle the complexity of the model. This, however, led to the introduction of several constant factors in different parts of the algorithms. In this paper we study four distributed node coloring algorithms in a more practical setting. We use the network simulator Sinalgo [1] to execute the algorithms in a variety of deployment scenarios in the static and the dynamic setting.

Let us briefly consider the algorithms we evaluate in this paper. All algorithms are designed for the realistic SINR model of interference and can be used to establish a TDMA schedule using methods from [4, 8]. We denote the number of nodes by n and the maximum degree in the network by Δ . The first algorithm, which we denote by MWCOLOR in this paper, was proposed by Moscibroda and Wattenhofer in [3] for the protocol model and transferred to the SINR model by Derbel and Talbi [4]. MWCOLOR computes an $\mathcal{O}(\Delta)$ -coloring in $\mathcal{O}(\Delta \log n)$ time slots by first selecting leader nodes, which coordinate the color selection of other nodes so that only few nodes in each neighborhood compete for the same color. As second algorithm we evaluate is a variant of the distributed $(\Delta + 1)$ -coloring algorithm proposed by Yu et al. [5]. Their algorithm computes leaders, which then increase their transmission power to block nodes that could hinder the leaders (original) neighbors from selecting a valid color. We use a variant of their original algorithm, as their algorithm operates in a slightly different setting, however, the ideas are applicable in our setting as well. We denote the variant by YUCOLOR and introduce it in Section 2.4. Finally, we consider two algorithms Fuchs and Prutkin proposed in [2, 9]. RAND4DCOLOR is a very simple randomized coloring algorithm that computes a (4Δ) -coloring by simply selecting a new random color whenever a conflict is detected. Finally, COLORRED, uses an existing coloring to coordinate the color selection process to compute a $(\Delta + 1)$ -coloring. The nodes executing the algorithm first select a set of leaders, which compute a medium access schedule based on the existing colors. Due to this schedule only few nodes are active at a time, which enables the active nodes to quickly win the competition for their final colors. All evaluated algorithms distributively compute the valid node coloring in $\mathcal{O}(\Delta \log n)$ time slots.

1.1 Related Work

In wireless networks, node colorings are particularly interesting as they allow the nodes to establish more efficient communication for example by computing a TDMA communication schedule (still using the realistic SINR model) [4, 8]. Although distributed node coloring algorithms are widely applicable and their study was initiated more than 25 years ago, only few experimental evaluations are concerned with distributed node coloring algorithms and none consider distributed algorithms for wireless networks. To the best of our knowledge the first experimental evaluation on distributed node coloring algorithms is due to Finocchi, Panconesi, and Silvestri [10]. They study very simple

node coloring algorithms, which are similar to our RAND4DCOLOR coloring algorithm. Their algorithms and the evaluation are based on simpler message-passing models, which do not consider interference. They observed that such simple algorithms are very fast, and proposed some practical improvements to the coloring algorithms. In an earlier experimental study Marathe, Panconesi and Risinger [11] considered simple edge colorings of the same randomized trial-and-error flavour as the later considered vertex coloring algorithms and found that such algorithms performed “extremely good”. Pindiprolu and Kothapalli [12] extend the study on distributed node coloring algorithms by Finocchi, Panconesi and Silvestri by considering the same randomized algorithms and compare it to a similar algorithm that requires only $\mathcal{O}(\sqrt{\log n})$ rounds of transmitting a single bit. Hernandez and Blum [13] compare a distributed coloring algorithm inspired by Japanese tree frogs to the algorithms studied by Finocchi, Panconesi, and Silvestri, however, they focus on minimizing the number of colors used.

1.2 Contribution

First, we show that the very simple RAND4DCOLOR coloring algorithm is very fast, achieving a runtime an order of one magnitude faster than its direct competitor, the MW-coloring algorithm. Interestingly, the algorithm computes a valid (4Δ) -coloring in less time slots than required for one round of local broadcasting. We additionally show that our COLORRED algorithm is significantly faster than YUCOLOR, our variant of the $(\Delta + 1)$ -coloring algorithm by Yu et al.

Second, we propose heuristic improvements for COLORRED, MWCOLOR, and YUCOLOR. The improvements are inspired by RAND4DCOLOR and allow the nodes to decrease the number of time slots accounted for the transmission of a message. We show that they considerably improve the runtime while keeping the number of conflicts in the network close to zero.

Third, we study the correction variants in a network with mobile nodes and in a network in which a large fraction of the nodes start the algorithm after the remaining network has computed a valid coloring. We observe that RAND4DCOLOR is most robust against mobility of nodes. Regarding the late wake-up of some nodes, we show that simple measures are sufficient to make RAND4DCOLOR robust in this scenario.

Outline: The remainder of this paper is structured as follows. In the next section we describe and introduce the algorithms we consider in our experiments. In Section 3 we describe the simulator and the setting of our experiments before evaluating the algorithms in Section 4. We conclude this paper in Section 5.

2 Considered Algorithms

Node coloring is the problem of assigning a color to each node in the network such that no two neighbors have the same color. In distributed computing, a $(\Delta + 1)$ -coloring (which is always possible) is the ultimate goal, as it is NP-hard to color a graph with the minimum number of colors even in a centralized way [14]. We call the color of a node *valid* if no neighbor selected the same color and say that a node that selected the same color as one of its neighbors to have a *conflict* with this neighbor. We use local broadcasting [15] as the basic form of communication, which allows successful communication within DURATION time slots (we determine this parameter in Section 4). We illustrate the flow of each algorithm on an example network in Appendix C.

2.1 RAND4DCOLOR

RAND4DCOLOR is based on a very simple randomized algorithm well-known for message-passing models since decades [16, Chapter 10]. It has recently been proven to be efficient in the SINR model by Fuchs and Prutkin [2]. The phase-based algorithm computes a valid $(\Delta + 1)$ -coloring in $\mathcal{O}(\Delta \log n)$ time slots. During each phase of the algorithm, the node may receive the colors of some of its neighbors. If, at the end of the phase, a conflict between its current color and the color received by a neighbor is detected, the node selects a new color at random. The overall execution is exactly as known from the message-passing models, however, the runtime is competitive by reducing the length of the phases. This leads to uncertainty in whether a conflict can be detected within each phases, which is bounded by Fuchs and Prutkin.

Apart from RAND4DCOLOR we consider some variants of the algorithm. For the first variant, RAND4DRESPCOLOR, we add a listening phase to the start of the algorithm and require the node to store the latest received color of each neighbor. When resetting the color the nodes do not select a color that is currently stored for a neighbor and thereby reduce the number of conflicts (especially in case of highly asynchronous wake-up of nodes). We do also consider a variant RAND4DFINALCOLOR that finalizes the selected color after it did not receive a color conflict for at least DURATION time slots. This enables each node of the algorithm to decide when the coloring algorithm is completed. In another variant we reduce the number of available colors to $\Delta + 1$ and call the variant RAND1DCOLOR. This variant computes a $(\Delta + 1)$ -coloring, although our theoretical guarantees holds only for $c > 4\Delta$ colors.

2.2 COLORRED

The algorithm COLORRED by Fuchs and Prutkin [2] computes a valid $(\Delta + 1)$ -coloring in $\mathcal{O}(\Delta \log n)$ time slots. COLORRED computes two levels of MISs, the first level MIS determines independent leaders, which then coordinate the activity of the remaining nodes. After the first level MIS all non-leaders request an active interval from the leaders, during which the non-leader nodes then repeatedly execute the faster second level MIS algorithm. Once a non-leader node v is in the independent set, it selects a color from the set of free colors F_v , transmits the color to all its neighbors and resigns from the independent set. The active interval is based on the initial (valid) color of the non-leader node and the schedule determined by the leader (independent from other leaders). These schedules achieve that few nodes compete in the second level MIS, which allows making these MISs very fast. We additionally consider a variants of COLORRED that does not use a valid node coloring but each node simply selects a random number from the set of available colors. We denote this variant by CRRANDCOLOR.

2.3 MWCOLOR

We implement the MW-Coloring algorithm as described by Derbel and Talbi [4] and denote it by MWCOLORING. The algorithm computes an $\mathcal{O}(\Delta)$ -coloring and proceeds as follows: First the nodes compete to be in an MIS to become leaders and select color 0. The remaining nodes request a continuous block of colors from a selected leader. Once this color block is received, the node competes in another MIS against at most constantly many neighbors for a color. If the MIS is won, it selects the color, otherwise it moves on to the next color in its color interval and competes again. A notable difference between COLORRED and MWCOLOR is that the number of time slots required for the color-competing MIS in MWCOLOR is significantly higher than the second level MIS in COLORRED, however, a lot less of these slower MISs are executed.

2.4 YUCOLOR

The coloring algorithm by Yu et. al [5] computes a $(\Delta + 1)$ -coloring in $\mathcal{O}(\Delta \log n + \log^2 n)$ time slots. The main idea behind achieving $\Delta + 1$ colors in their algorithm is to increase the transmission power in order to coordinate the color selection process within a larger distance. To achieve this the algorithm uses two transmission powers r_1 and r_2 , where r_1 is the regular broadcasting range and $r_2 = 3 \cdot r_1$. The algorithm itself works as follows: First, the nodes compute an MIS with respect to r_2 . All nodes in the MIS transmit a so-called DoNotTransmit-message to all nodes within r_2 . Thereby the nodes within the range r_2 enter a blocked state S, which they only leave once they receive a StartTransmit-message or a StartColoring-message. The nodes in the MIS transmit a StartColoring-message, however, only to the nodes within the smaller range r_1 . These nodes start with the color selection process by transmitting an AskColor-message to their MIS node. The MIS node coordinates the requests and allows one after the other to select the smallest color not taken by a neighbor. The colors can be selected without a conflict, as all close-by nodes are either coordinated by the MIS node or are in the blocked state S. Naturally, once a color is selected by a node, the node informs all its neighbors about its color selection.

In the setting Yu et. al designed the algorithm for, the nodes are not given an estimate of the maximum degree Δ . Thus, to ensure successful communication a slow-start mechanism is used for the transmissions. To circumvent this length mechanism we adapt the algorithm to the case of known Δ and prove the following theorem.

Theorem 1. *YUCOLORING computes a $(\Delta + 1)$ -coloring in $\mathcal{O}(\Delta \log n)$ time slots in our setting.*

The correctness essentially follows from the correctness of the original algorithm. For the argument to be more concise, we elaborate on the main points in the following and give a pseudocode of YUCOLOR in Algorithm 1.

The coloring is valid: Let us consider a node v and assume its coloring is not valid due to a conflict with its neighbor u . If v has color 0, it is a leader node and the conflicting node u must have been one of the nodes v dominated. Thus, with high probability, u received the DoNotTransmit and the StartColoring, afterwards transmitted AskColoring itself and received a Grant message - leading to the selection of another color $c \neq 0$. If v 's color is not 0, it selected its color during such a color selection process itself. As both v and u successfully transmit their color after selection with high probability and neighbors respect this selection, it must be the case that v and u selected the color simultaneously. Since the leader nodes wait long enough between transmitting the Grant message to the two nodes, this can only happen if v and u listen to two different leaders. This, however, is not possible as all nodes within at least two broadcasting ranges of v received the DoNotTransmit message of v 's leader with high probability.

All nodes get colored: Essentially this holds as each node v is either in the MIS (with respect to r_2) at some point or one of its neighbors is in the MIS and allows v to select a color.

The runtime of the algorithm is $\mathcal{O}(\Delta \log n)$ time slots: Let us consider the maximum time until a node v or one of its neighbors is in the MIS. Remember that the MIS is computed with respect to the range r_2 , while the neighborhood relation we consider for the coloring is relative to r_1 . As each r_1 -neighborhood of a MIS node is colored after $\mathcal{O}(\Delta \log n)$, this is also the asymptotic time that passes between the MIS node transmits DoNotTransmit and StartTransmit. As at most 36 nodes can be independent regarding the range r_1 in the r_2 -range of a node v , after at most 36 rounds of the MIS (and potentially the following blocked/coloring state) all nodes in the r_2 -range

Algorithm 1: YUCOLORING for node v

```
1 Continuously:
2 if Received DoNotTransmitu then  $F_v \leftarrow F_v \cup \{u\}$  and transit to  $state \leftarrow blocked$ 
3 if Received Coloru(c) from node u then  $C_v \leftarrow C_v \cup \{c\}$ 
4 switch  $state$  do
5   case start
6     wait for  $\mathcal{O}(\Delta \log n)$  time slots
7     transit to  $state \leftarrow MIS$ 
8   case blocked
9     if Received StartColoringu then transit to  $state \leftarrow C1$ 
10  case MIS
11    We use MIS( $\ell = 1$ ) from COLORRED [2]. Successful nodes transit to  $state \leftarrow leader$ 
12  case leader
13    Transmit DoNotTransmitv with range  $r_2$  and prob.  $p_{high}$  for  $\mathcal{O}(\log n)$  slots
14    select color 0
15    Transmit StartColoringv with range  $r_1$  and
16    prob.  $p_{high}$  for  $\mathcal{O}(\log n)$  time slots
17    if  $Q$  not empty then
18       $u \leftarrow Q.pop()$ 
19      Transmit Grantu with range  $r_1$  and prob.  $p_{high}$  for  $\mathcal{O}(\log n)$  time slots
20    else
21      Transmit StartColoringv with range  $r_1$  and
22      prob.  $p_{high}$  for  $\mathcal{O}(\log n)$  slots
23  case C1
24    Transmit AskColorv with range  $r_1$  and
25    prob.  $p_{low}$  for  $\mathcal{O}(\Delta \log n)$  time slots
26    if Received Grantu then transit to  $state \leftarrow C2$ 
27  case C2
28    select smallest color  $c$  not in  $C_v$ 
29    Transmit Colorv( $c$ ) with range  $r_1$  and prob.  $p_{high}$  for  $\mathcal{O}(\log n)$  time slots
```

of v must have either been in the MIS or are neighbors of an MIS-node. Thus, either v or one of its neighbors wins the MIS competition and starts the coloring routine afterwards. Overall, this results in a runtime of $\mathcal{O}(\Delta \log n)$ time slots.

2.5 Correcting Variants

Apart from RAND4DCOLOR, the coloring algorithms do not account for errors, as they do not happen if all transmissions are successful. We denote the number of time slots required to ensure successful transmission by all nodes by DURATION, but even this does not guarantee that there are no failures (due to the probabilistic nature of the transmissions). Thus, we consider so-called *correcting variants* of the algorithms, in which we combine the algorithms with ideas of RAND4DCOLOR. Namely, we decrease the time accounted for successful transmission of messages, and resolve the introduced conflicts afterwards. We resolve the conflicts by resetting non-leader nodes to the last uncolored state that is still proper. For COLORRED nodes compute a new valid active interval based on the previous active interval and the schedule length and wait for this new interval. In MWCOLOR the nodes reset to the first color competition of the color block they received. For YUCOLOR the nodes must reset the initial MIS, as the nodes leader might have resigned by now. For leader nodes we use the simpler strategy of resetting to a random color to prevent issues that arise once leaders may resign from their duties. For COLORRED we use the set of unused colors F_v and for MWCOLOR and YUCOLOR we use $[\Delta]$. We denote the correcting variants by CRRCOR, MWCOR, and YUCOLOR.

2.6 Determining Parameters

There are only few parameters of the algorithms that must be determined apart from those related to communication. Let us therefore first consider these parameters. Except for RAND4DCOLOR all algorithms use local broadcasting and fast local broadcasting. While local broadcasting might be used by all nodes simultaneously, fast local broadcasting is restricted (by theoretical considerations) to a constant number of nodes in each broadcasting range. We determine the optimal transmission probability and the duration (denoted by DURATION) of local broadcast in Section 3.1 and determine a factor (denoted by FACTOR) between local broadcasting and its fast variant for the algorithms. Although RAND4DCOLOR does not use regular broadcasting, we still use FACTOR to determine the length of the phases (this is not the same but related to a fast local broadcast). Apart from this, there are no parameters required for RAND4DCOLOR and YUCOLOR. For CRRCOR and MWCOLOR, one could set additional parameters that determine the time slots accounted for a second level MIS and the length of a color block, respectively. We do not set those parameters but set the length of a second level MIS as a fast local broadcast, and the color block to be 8 (as we do not rate the algorithms based on the number of colors they require, using a large enough value is sufficient).

3 Sinalgo Settings

We conduct our experiments using the current version 0.75.3 of Sinalgo [1], an open-source simulation framework for networks algorithms in Java. Sinalgo has built-in support for a variety of communication and interference models, and is implemented in a modular fashion, making it easy to add customized models or algorithms. The main simulation framework offers both round-based and asynchronous or event-based simulation. Apart from that connectivity, interference, mobility, reliability, distribution, and message transmission models implement a wide variety of settings.

We shall briefly introduce the relevant parts of the simulation framework in the following. In the round-based setting, in every time slot each node is considered once. Thus, the node handles all successfully received messages and performs one step. Both the message handling and the step depend on the implementation of the network algorithm. In the event-based simulation, each action of the algorithm must be invoked by a timer. Despite the absence of rounds or slots, we denote the time required for one transmission as a time slot. Mobility is not supported in this setting due to the high number of events and the corresponding updates of all positions (required for example for connectivity and interference computations). Let us now consider the models used in our experiments.

Connectivity: To determine the neighborhood relations we implemented a new model, which calculates the broadcasting range directly based on the SINR parameters used in the simulation. Based on the broadcasting range, the neighbors of a node v are determined as the nodes within this range of v .

Interference: Successful reception of the nodes transmission is determined by the standard geometric SINR model in all our simulations. In the SINR model a transmission from a sender to a receiver is *feasible* if it can be decoded by the receiver. It depends on the ratio between the desired signal and the sum of interference from other nodes plus the background noise whether a certain transmission is successful. Let each node v in the network use the same transmission power P . Then a transmission from u to v is feasible if and only if

$$\frac{\frac{P}{\text{dist}(u,v)^\alpha}}{\sum_{w \in I} \frac{P}{\text{dist}(w,v)^\alpha} + N} \geq \beta,$$

where $\alpha \in [2, 6]$ is the attenuation coefficient, the constant $\beta > 1$ depends on the hardware, N denotes the environmental noise, $\text{dist}(u, v)$ the Euclidean distance between two nodes u and v , and $I \subseteq V$ is the set of nodes transmitting simultaneously to u . The parameters for the geometric SINR model of interference are set as follows. We use a value of $\alpha = 4$ for the attenuation coefficient, which is assumed to be between 2 for a free field environment and 6 for buildings in practice [17]. We use a threshold of $\beta = 10$ and an environmental noise value of 10^{-9} , which are both within the ranges often reported [18, 19]. We use a uniform transmission power, which is set to $P = 1$ for all nodes. The *broadcasting range* r^B of a node v defines the range around v up to which v 's messages should be received. Based on the SINR constraint, the *transmission range* $r^T \leq (\frac{P}{\beta N})^{1/\alpha}$ is an upper bound for the broadcasting range (with $r^B < r^T$ to allow multiple simultaneous transmissions). Our parameters lead to a transmission range of 100 m, an additional broadcasting range parameter of 2 leads to the broadcasting range of about 84 m.

A flaw in the SINR-module delivered with Sinalgo in version 0.75.3 drops some transmissions although they are feasible in the SINR model. We show how to correct this in Appendix E. For more details on the SINR model itself we refer, for example to [2].

Distribution: We mostly deploy our nodes on a square area of $1000 \text{ m} \times 1000 \text{ m}$ using several distribution strategies. We use the built-in random and grid model, which deploy the nodes uniformly at random and according to a regular two-dimensional grid, respectively. Custom models we use are a perturbed grid, in which each nodes grid position is uniformly at random drawn from a 1 m^2 area centered at the original grid position, and a cluster distribution which distributes all nodes in a predefined number of clusters (we use 10 clusters). Additionally, we use a combination of the cluster model with the other models, in which 50 % of the nodes are distributed according to the cluster model and the remaining nodes according to either the random, grid or perturbed grid model. Our distribution models are illustrated in some example networks in Fig. 4 in Appendix A. To increase comparability of our experiments we use a precomputed set of position files.

Message transmission: Messages are transmitted with a certain probability in each time slot. We compute the transmission probability based on a transmission constant TXCONST and the maximum degree Δ in the network as $\text{TXCONST}/\Delta$ (apart from constant factors this is as described by Goussevskaia et al. [15]). Regarding the time required for a message transmission we assume constant time message transmission. As all messages are of size at most $\mathcal{O}(\log n)$ in our algorithms, this fits the algorithms requirements. To ensure that a message can be transmitted in one time slot, which is of length 1, we use a transmission time of 0.999 time slots. We do neither allow simultaneous transmission and reception nor the simultaneous reception of several packets.

Reliability: Throughout our simulations we use so-called reliable transmission, which implies that transmitted messages are received unless they are discarded by the interference model.

Mobility: Sinalgo supports mobility based on either random waypoints, or random directions. We use the latter model, as it consistently provides a balanced distribution of the nodes on the area, while the random waypoint model would lead to a high concentration of nodes in the center of the deployment area [20].

Our algorithms are implemented in subclasses of the node class, which plugs into the described models and provides standard transmission and reception features. To measure the number of nodes with a valid and an invalid color, each node notifies the simulator whenever it selects a new color, which is then checked for validity with colors selected by the neighbors. This is done within the simulation framework, thus we do neither use messages nor tell the nodes about the result of this color inspection.

For each experiments we use 100 runs on the same pre-computed deployments. Apart from the overall results we report only results for the random deployment due to space constraints, other results are deferred to Appendix E. We measure the time required to compute a valid coloring and the number of nodes that were not able to select a valid color. We use one time slot as the time required for one transmission. To measure the runtime of our algorithms, we deploy the nodes simultaneously in the area and start the algorithms asynchronously after a waiting period that is chosen uniformly at random between 0 and 10 time slots (using real numbers) for each node. The runtime measurement starts with the deployment of the nodes and ends once all algorithms are in a finished state or all nodes have selected a valid color. Note that the time slots of the nodes are not synchronized and may overlap partially.

3.1 Basic Communication Parameters

To implement the message transmission in the algorithms, we use local broadcasting with known Δ , and therefore use the parameters transmission constant TXCONST and broadcast duration DURATION . To compute the transmission probability used by the nodes we divide a parameter TXCONST by the maximum degree Δ . Other parameters are the DURATION , which is set to the number of time slots required for reliable communication in the network, and FACTOR , which is part of the ratio between regular and fast local broadcasting: $\Delta \cdot \text{FACTOR}$. Although RAND4DCOLOR does not use local broadcasting we use the same TXCONST , to increase comparability of the algorithms. We show the relation of the parameters in calculating the values used for the simulation in Table 1

To determine the parameter TXCONST and DURATION we execute local broadcasting for several values of TXCONST and report the optimal parameter and the runtime in Table 2. Additionally we report some characteristics of the networks such as the average maximum degree Δ and the average degree, the transmission probability (which is computed as $\text{TXCONST}/\Delta$). Note that we set DURATION so that it is slightly larger than the average maximum runtime of local broadcasting. This should achieve successful transmission with a high probability.

Table 1: Transmission probabilities and durations based on the simulation parameters

Simulation parameter	Value
Local broadcast	
transmission probability	$\frac{\text{TXCONST}}{\Delta}$
transmission duration	DURATION
Fast local broadcast	
transmission probability	$\frac{\text{TXCONST}}{\Delta} \times \Delta \times \text{FACTOR}$ $= \text{TXCONST} \times \text{FACTOR}$
transmission duration	$\frac{\text{DURATION}}{\Delta \times \text{FACTOR}}$

Table 2: Parameters that achieve successful local broadcasting in the different distributions. R=Random, G=Grid, PG=PerturbedGrid, C=Cluster

Distribution	R	G	PG	C	C&R	C&G	C&PG
TXCONST	0.15	0.15	0.10	0.30	0.25	0.20	0.20
Maximum degree Δ	36.6	20.0	27.9	182.0	106.0	101.8	100.8
Average degree	20.6	18.6	20.9	93.8	39.4	38.7	39.9
Transmission probability	4.11×10^{-3}	7.5×10^{-3}	3.59×10^{-3}	1.71×10^{-3}	1.46×10^{-3}	2.02×10^{-3}	2.55×10^{-3}
Avg. runtime of a local broadcast	4592	3345	4845	12 903	8062	8183	8089
DURATION	4600	3400	4900	12 900	8100	8200	8100

4 Experiments

In this section we evaluate the distributed node coloring algorithms described in Section 2 using the simulation framework described in Section 3. Therefore we determine a final parameter FACTOR, which determines the difference between regular local broadcasting and a fast local broadcasting regarding both the duration and transmission probability. Afterwards we consider the algorithms and their variants separately in Sections 4.2 to 4.4. In Section 4.5 we compare the progress of the algorithms before comparing the algorithms themselves in Section 4.6. In Section 4.7 we study the performance of the algorithms under network dynamics such as moving nodes and the highly asynchronous wake-up of nodes.

4.1 Determining FACTOR

We use the parameters TXCONST and DURATION to achieve reliable local broadcast. In this section we determine the parameter FACTOR, which we use to increase the transmission probability and decrease the time accounted for the *fast* local broadcast (by multiplying and dividing by $\Delta \cdot \text{FACTOR}$, respectively).

In contrast to the other coloring algorithms that we evaluate, RAND4DCOLOR does not use regular local broadcasting but only a variant that achieves success with a certain probability. Therefore, we use $\text{DURATION} \times \text{FACTOR}$ as the length of each phase, while using the regular TXCONST from local broadcasting, as all nodes transmit during each phase. We study the length of the phases in Appendix B and observe that the shorter the phases, the faster the algorithm (without any disadvantages). Thus, we use $\text{FACTOR} = 0.001$ in the following. For COLORRED, MWCOLOR, and YUCOLOR, $\Delta \cdot \text{FACTOR}$ is the ratio between local broadcasting with all nodes and fast local broadcasting with a small subset of the nodes. As the optimal value of FACTOR may depend on the size of the sets, how often this mode of transmission is used, and possibly other factors, we

determine this parameter for each of the algorithms separately. The results using 1000 nodes and the random deployment scheme are given in Table 3.

Table 3: Average number of conflicts and average runtime using different parameters FACTOR.

FACTOR		0.05	0.1	0.2	0.3	0.4	0.6	0.8
COLORRED	conflicts	0.00	0.04	0.10	0.00	0.12	0.51	2.47
	runtime	339 013	171 099	87 924	59 995	46 266	32224	25 384
MWCOLOR	conflicts	0.1	0.1	0.4	1.0	1.5	2.7	-
	runtime	81 195	44 700	27982	23 995	22 807	21 870	-
YUCOLOR	conflicts	0.6	0.7	1.4	2.9	6.4	22.4	-
	runtime	286 167	160 088	99946	82 707	72 660	67 131	-

We observe that the number of conflicts increases with the parameter FACTOR, while the runtime decreases. We select the parameter FACTOR = 0.6 for COLORRED and FACTOR = 0.2 for MWCOLOR and YUCOLOR. This results in an average number of conflict of 0.51, 0.4 and 1.4 after an average runtime of 32 224, 27 982 and 99 946 time slots for COLORRED, MWCOLOR and YUCOLOR.

4.2 RAND4DCOLOR and its Variants

Let us now compare the different variants of RAND4DCOLOR we described in Section 2.1. We use FACTOR = 0.001 as determined previously. The result of this comparison is given as Table 4 for the random distribution. Clearly, the basic RAND4DCOLOR algorithm is the fastest with a runtime of only 1256 time slots. This was expected as the variants either improve the resulting coloring or make the algorithm more robust regarding a specific setting. For RAND4DRESPCOLOR the nodes wait DURATION time slots before selecting a color in order to learn colors already selected by neighbors, while RAND4DFINALCOLOR waits for DURATION time slots after it detected the last conflict before finalizing its color. Thus both variants require approximately the runtime of RAND4DCOLOR plus DURATION: 5668 and 5865 time slots, respectively. RAND1DCOLOR reduces to set of available colors, which leads to more conflicts. Thus the runtime of 4174 time slots is surprisingly good. For these algorithms the number of finished nodes corresponds to the number of nodes with a valid color, as these variants do never finalize their color. Only for RAND4DFINALCOLOR the value corresponds to the number of nodes that finalized their color.

4.3 Does COLORRED Require Valid Colors?

The initial color of COLORRED must correspond to a valid node coloring (which is pre-computed at no cost). The variant CRRANDCOLOR, however, simply draws a random number from the a set of colors. We study in this section whether this potentially invalid initial coloring can still be used to compute a valid $(\Delta + 1)$ coloring. We measure the average runtime and number of conflicts for the sets $\{0, 1, \dots, c \cdot \Delta\}$, with $c = 1, 2, 3$ ¹. We report the results in Table 5 for the random deployment.

The results indicate that COLORRED does not require a valid coloring to perform well in practice. We see a significant increase in the number of conflicts for CRRANDCOLOR only for colorings of cardinality $\Delta + 1$, however, assuming a valid $\Delta + 1$ coloring to be given renders executing the algorithm unnecessary. For colorings of size larger than 2Δ the difference in the number of conflicts and the runtime of the algorithms is negligible. Also, we observe that the smaller the color set the

¹For $c > 1$ we write $c\Delta$ instead of $c(\Delta + 1)$ for brevity.

Table 4: Comparison of average runtime and average number of conflicts of our RAND4DCOLOR variants

	runtime	conflicts
RAND4DCOLOR	1256	0.00
RAND4DRESPCOLOR	5668	0.00
RAND4DFINALCOLOR	5865	0.00
RAND1DCOLOR	4174	0.00

Table 5: Comparing COLORRED and CRRANDCOLOR for a varying number of colors in the initial coloring.

Number of initial colors	$\Delta + 1$	2Δ	3Δ
COLORRED			
conflicts	3.29	0.86	0.55
runtime	18 638	24 824	32 197
CRRANDCOLOR			
conflicts	6.52	0.93	0.65
runtime	19 766	24758	32 287

faster the algorithm. As computing a valid coloring additionally requires some effort, we focus on the variant CRRANDCOLOR with a random (2Δ)-coloring in the following.

4.4 Correcting Variants

In this section we consider heuristic improvements to CRRANDCOLOR, MWCOLOR, and YUCOLOR, namely CRRCOR, MWCOR, and YUCOR. With these heuristics, we aim at making the algorithms both faster and more robust towards failures in the communication. Instead of ignoring detected color conflicts as it is mostly done in the basic algorithms, we actively deal with them and try to resolve the conflicts, cf. Section 2.5. It is obvious that, although the number of conflicts may be reduced, the runtime of the algorithm increases for these variants. However, as the algorithms are able to detect and resolve conflicts, we reduce the time accounted for a successful transmission to decrease the runtime while introducing some (hopefully temporary) conflicts. We do this by reducing the parameter DURATION to a fraction of the value and denote the reduced parameter by DURATION'. We report the results for the random deployment in Table 6.

Table 6: Average runtime and conflicts by the correcting variants. We used varying fractions of the DURATION parameter (denoted by DURATION') and the random deployment.

Fraction of DURATION Resulting DURATION'		1/32	1/16	1/8	1/4	1/2	1
CRRCOR	conflicts	0.00	0.00	0.00	0.00	0.00	0.00
	runtime	8965	6984	6489	8883	14 348	25 218
MWCOR	conflicts	0.13	0.23	0.10	0.02	0.02	0.00
	runtime	11 065	7688	6834	9105	15 762	31 027
YUCOR	conflicts	4.62	1.23	0.28	0.09	0.00	0.00
	runtime	4370	9807	16 652	29 635	58 189	116 646

For the standard case of unchanged DURATION we observe that the average number of conflicts decreases from 0.93, 0.4 and 1.4 (cf. Tables 3 and 5) to 0.00 for CRRCOR, MWCOR and YUCOR, while the runtime increases as expected. Using a smaller parameter DURATION', however, the correcting variants are able to improve upon the basic algorithms. For CRRANDCOLOR the correcting variant achieves to compute a coloring without conflicts even for very small values of DURATION'. The best runtime is obtained using DURATION' = 575, for which the algorithm computes a $(\Delta + 1)$ -coloring in 6489 time slots. Similarly, MWCOR achieves to $\mathcal{O}(\Delta)$ -color the network even for the smallest considered DURATION' values essentially without a conflict. The (very) small number of average conflicts we observed is probably due to not-yet detected conflicts. As MWCOLOR does not store the neighbors colors new conflicts occur more frequently than in CRRCOR. The best runtime of MWCOR is also achieved for DURATION' = 575, resulting in 6834 time slots. YUCOR, on the

other hand, does not achieve a coloring without conflict for the smaller DURATION' values. This is due to the blocking of nodes due to the DoNotTransmit messages. For small DURATION' values, some nodes are not able to receive the StartTransmit message and remain blocked for remaining algorithm. The best runtime is achieved for DURATION' = 287 with only 4370 time slots, however, leading to an average number of conflicts of 4.62. Thus we select DURATION' = 287, resulting in an $(\Delta + 1)$ -coloring algorithm with an average number of 1.23 conflicts after 9807 time slots (which is still an order of magnitude faster than the basic algorithm).

4.5 Comparing the Progress of the Algorithms

Before comparing the runtime of all algorithms, we briefly study the progress of RAND4DCOLOR, the algorithms CRRANDCOLOR, MWCOLOR, and YUCOLOR, and their correcting variants. The average progress (i.e., the average number of finished nodes over time) of all algorithms is illustrated in Fig. 1.

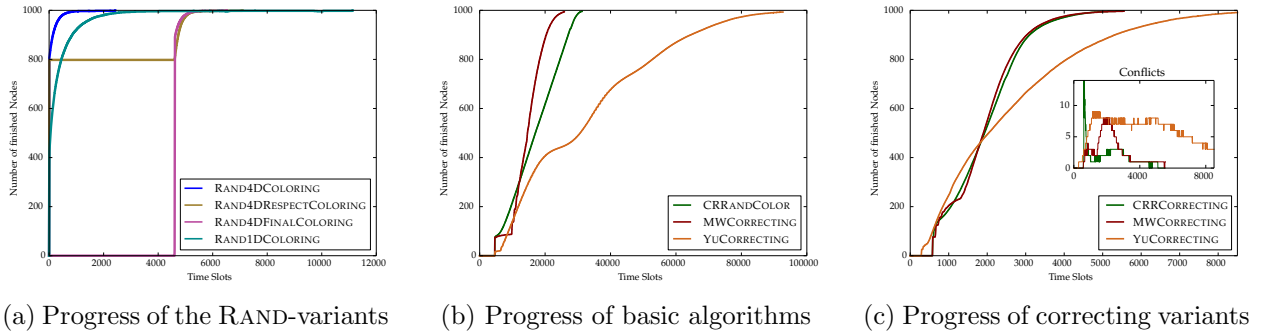


Figure 1: Progress of the algorithms COLORRED, MWCOLOR, YUCOLOR (left) and their correcting variants (right). For the correcting variants we additionally depict the average number of conflicts.

We observe in Fig. 1a that while RAND4DCOLOR and RAND1DCOLOR are able to make fast progress after the algorithms started, the two remaining variants do not resolve a conflict or finalize a color for a little more than 4000 time slots before finishing within the next 1000-2000 time slots. This behaviour is due to two reasons. For RAND4DRESPCOLOR, which is expected to be more robust in the case of heterogenous wake-up patterns, this is caused by a listening period, during which the colors of neighbors are received and stored. Afterwards, this variant selects its color so it does not coincide with the latest received color from any of the neighbors. For RAND4DFINALCOLOR it is due to a waiting period before finalizing the current color. This waiting period of a node is reset with each conflict it detects and allows the node to decide when a selected color can be finalized. In both algorithms the waiting or listening period is set to exactly DURATION time slots. Finally, RAND1DCOLOR decreases the number of used colors to $\Delta + 1$, resulting in a runtime of 4174 time slots. Note that reducing the number of colors to $\Delta + 1$ results in a higher variance in the runtime, as the probability to select a color of one of the neighbors increases.

Regarding the remaining algorithms, the first, sudden increase is exactly after DURATION (or DURATION') time slots. This is when the first nodes enter the MIS, become leader and select their color. As the MIS is computed regarding a three times larger broadcasting range in YUCOLOR, a lot less nodes enter the MIS in this algorithm. Some time after the MIS computation, the nodes around the leaders select their colors. While this allows both CRRANDCOLOR and MWCOLOR to gradually color all nodes in the network, several plateaus can be observed for YUCOLOR. This is due to the fact that in YUCOLOR not all nodes are able to select a color after the first MIS execution, as the MIS is computed regarding a larger broadcasting range than the neighborhood

around the leaders which is allowed to select a color. Thus, we see at least two more MIS executions leaving their traces in this progress around time slots 20 000 and 40 000 for YUCOLOR in Fig. 1b. Although this happens also in YUCOR, we cannot clearly observe the moment it happens in the average progress depicted in Fig. 1.

For CRRANDCOLOR and MWCOLOR, the leaders dominate the whole network, resulting in each node requesting an active interval or a color interval from its respective leader. Let us compare the progress of CRRANDCOLOR and MWCOLOR. Recall that leaders in CRRANDCOLOR coordinate the time interval in which dominated nodes compete to select a final color, while in MWCOLOR the leaders coordinate which colors the dominated nodes compete for. This fundamental difference allows CRRANDCOLOR to use fast local broadcasting for the second level MIS (which can be seen as a competition for being allowed to select a valid color). The coordination of active nodes based on the nodes' initial color leads to an almost perfectly linear increase of the number of finished nodes over time. In MWCOLOR, the nodes compete in fewer but slower MIS-executions for the colors, which leads to the rapid increase once the threshold is reached (in Fig. 1b at around 9000 time slots). Note that the rapid increase once the threshold is met is due to the requirement of achieving a valid node coloring with very few conflicts, however, it also hints that faster progress is possible, as shown by our correcting variants.

Regarding our correcting variants, we observe that the progress of CRRCOR and MWCOR is very similar. One reason for this is that both use the same DURATION' value of 575 time slots. The second reason is that by reducing the time accounted for local broadcasting, all the slack is removed from the algorithms. As both algorithms elect leaders, which then allow a dominated node to either be active or compete for certain colors (depending on the algorithm), the remaining progress essentially shows this similarity.

Inside Fig. 1c we additionally show the number of conflicts occurring in the respective algorithms over time. We observe that in general, the number of simultaneous conflicts is relatively low with well below 15 conflicts at each time. For CRRCOR there is a peak at or around the leader election phase, indicating that too many nodes entered the MIS. A smaller peak is also visible for MWCOLOR, however, here more nodes fail to select a valid color in the following color competition. For YUCOR, the number of conflicting nodes is relatively stable at around 8 simultaneous conflicts. Although the number of conflicts decreases, not all can be corrected, as some conflicts are due to blocked nodes².

4.6 Performance Comparison of Coloring Algorithms

To compare the performance of the algorithms on the different deployment strategies, we show the runtime and the number of conflicts of the algorithms on the different deployments in Table 7. For the random deployment, the values are the best values from Tables 3 to 6. For the remaining distributions we selected the values analogously, cf. Appendix E for detailed results. (To get even more insight in the behaviour of the algorithms we study their progress, also in the full version.)

Regarding the conflicts we tried to select the parameters so that the number of conflicts are low. Only for YUCOLOR and YUCOR we allowed a slightly higher number of average conflicts to reduce the runtime, if possible. Thus, we focus on the runtime in the following. The results of the algorithms are very consistent throughout the different deployments. This indicates that the communication parameters are sufficiently well chosen to allow the algorithms to deliver their performance without being constrained by, for example, congestion problems.

²Recall that we set the leader to a quit-state and select color 0 once the leader of a blocked node resigns from its leader functionality (cf. Section 2.4).

Table 7: Average runtime and number of conflicts for all considered algorithms in all considered deployment strategies.

Distribution		R	G	PG	C	C&R	C&G	C&PG
RAND4DCOLOR	conflicts	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	runtime	1256	974	1372	3321	2316	2186	2016
RAND1DCOLOR	conflicts	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	runtime	4174	3358	4548	11 822	8153	8211	6627
CRRANDCOLOR	conflicts	0.93	0.82	0.10	1.05	1.83	1.03	3.23
	runtime	24 758	20 367	27 817	67 881	42 034	42 692	42 385
CRRCOR	conflicts	0.00	0.00	0.00	0.00	0.00	0.00	0.02
	runtime	6489	7017	7017	17 802	11 884	11 333	10 896
MWCOLOR	conflicts	0.42	0.26	0.28	0.56	0.44	0.34	1.06
	runtime	27 982	25 456	32 812	73 670	46 363	47 041	46 142
MWCOR	conflicts	0.10	0.21	0.12	0.45	0.44	0.36	0.42
	runtime	6834	5741	7567	23 531	12 780	13 535	13 353
YUCOLOR	conflicts	1.39	2.01	1.12	0.9	0.88	0.94	1.30
	runtime	99 946	113 105	129 054	164 839	122 267	141 003	126 488
YUCOR	conflicts	1.23	1.77	2.15	0.74	0.81	1.23	0.64
	runtime	9807	8654	11 479	20 849	15 060	15 835	14 620

We do not depict the basic variant COLORRED in Table 7 as the performance is essentially the same as CRRANDCOLOR (cf. Table 5). As CRRANDCOLOR and CRRCOR compute $(\Delta + 1)$ -colorings, YUCOLOR and YUCOR are its main competitors. A valid coloring of the same size is also computed by our variant RAND1DCOLOR that heuristically reduces the number of available colors in RAND4DCOLOR to $(\Delta + 1)$, however, we discuss this variant later. CRRANDCOLOR computes a $(\Delta + 1)$ -coloring using between 20 367 and 67 881 time slots. The correcting variant CRRCOR reduces the runtime to values between 6489 and 17 802 time slots. This is at par with MWCOLOR and MWCOR, and significantly less than YUCOLOR and YUCOR. The basic algorithm YUCOLOR requires between 99 946 and 164 839 time slots, while YUCOR reduces the runtime to values between 8654 and 20 849 time slots.

Our other algorithm, RAND4DCOLOR, computes a (4Δ) -coloring, hence MWCOLOR and its variant MWCOR are its main competitors. Depending on the deployment strategy, RAND4DCOLOR achieves a runtime between 974 and 3321 time slots. This is by far superior to the runtime achieved by both MWCOLOR and MWCOR. MWCOR requires between 5741 and 23 531 time slots, which is between 4 and 5 times the runtime of RAND4DCOLOR. MWCOLOR achieves a runtime between 25 456 and 73 670 time slots. Note that RAND4DCOLOR does not finalize the colors, however, even³ using a DURATION for finalizing the color, the variant RAND4DFINALCOLOR which finalizes colors, achieves a runtime between 4354 and 16 110 time slots (cf. Table 4).

Reducing the number of available colors in RAND4DCOLOR to only $(\Delta + 1)$ colors yields a $(\Delta + 1)$ -coloring heuristic, which we denote by RAND1DCOLOR. The heuristic selects a random color from $[\Delta]$ by resolving conflicts once detected and requires only between 3358 and 11 822 time slots. Thus it achieves the lowest runtimes of all considered $(\Delta + 1)$ -coloring algorithms. As mentioned, however, RAND4DCOLOR and its variants do not finalize their colors. Hence, if finalization of the colors is required CRRCOR achieves the best results for $(\Delta + 1)$ -coloring algorithms.

We conclude from our comparison that the best performance for $\mathcal{O}(\Delta)$ -colorings is achieved by RAND4DCOLOR and for $(\Delta + 1)$ -colorings by RAND1DCOLOR and CRRCOR, depending on the exact setting.

³Tentative experiments indicate faster runtimes if a reduced parameter DURATION' is used.

4.7 Coloring in Dynamic Networks

In this section we consider two scenarios, in the first one we allow the nodes to move, which forces the algorithms to maintain the validity of the coloring in a dynamically changing network. The algorithms cannot maintain the coloring valid at each node, as the neighborhood changes in an unforeseen manner. Thus, we study how large the fraction of nodes is that maintains a valid color despite the dynamic changes. In the second scenario a fraction of the nodes wake up after the remaining part of the network has already selected a color. Thereby we evaluate how well the algorithms can cope with highly asynchronous wake-up schemes. We restrict ourselves to the random deployment for these experiments and do not use pre-computed position files for the wake-up scenario.

In this section we consider only 250 nodes on a deployment area of $500\text{ m} \times 500\text{ m}$, which results in a similar density as in the previously considered settings. We reduce the number of nodes, as mobility increases the time required for the simulation significantly. Due to the high complexity of updating the relevant positions for each single event the used simulation framework Sinalgo additionally requires the synchronous mode, in which time slots of the different nodes are perfectly synchronized. Thus, it is sufficient to update the positions once per time slot. The nodes are deployed uniformly at random and move according to the RandomDirection model, cf. Section 3. The nodes alternate between moving and waiting time, both times are drawn randomly and follow a Gaussian distribution with mean value of 100 time slots and a variance of 50. The speed of a node is also drawn at random according to a Gaussian distribution with a mean between 0 and 1 meter per time slot and a variance of 2. For simplicity we refer to the mean values as the *node speed*.

We show the number of finished nodes over time for RAND4DCOLOR, CRRCOR, MWCOR, and YUCOR for node speed values of 0, 0.1, 0.5, and 1.0 in Fig. 2.

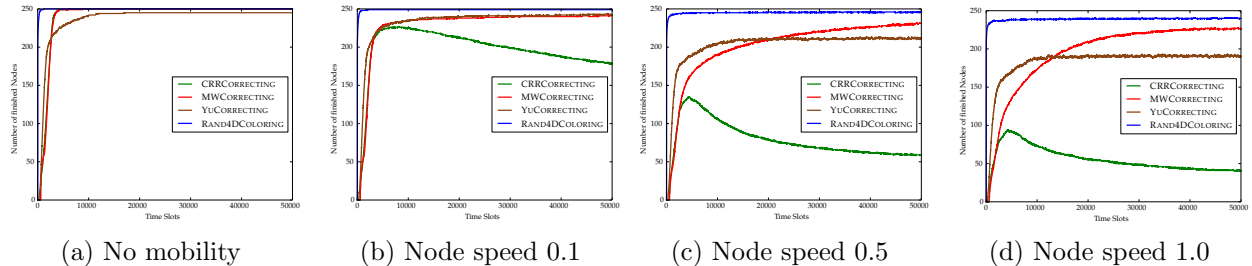


Figure 2: The progress of the coloring for the different algorithms. We depict the average number of finished nodes over time for different nodes speed values drawn according to a Gaussian distribution with mean values between 0 and 1.

RAND4DCOLOR, MWCOR, and YUCOR maintain a relatively high fraction of valid nodes, while CRRCOR performs poorly. The main reason for the poor performance of CRRCOR is that a relatively long time passes between detecting the conflict and selecting a new color. Conflicting non-leader nodes compute the next valid active interval based on the previous active interval and the schedule length. Due to mobility, however, the active interval does not guarantee that few nodes are active in each interval. Instead it mainly leads to many nodes waiting to compete for a color. This is different for both YUCOR and MWCOR, which reset to a state in which they can compete for a valid color right immediately. Thus, while the schedule helps coordinating the color selection scheme in the static setting, this is not the case for the dynamic setting.

YUCOR⁴, maintains a significantly higher fraction of valid colors than CRRCOR. Non-leader

⁴In this setting we do not select color 0 for blocked nodes, as nodes may leave the blocked state. As this does not

nodes may get blocked and separated from the blocking node, which could also lead to a low performance. However, the algorithm prevents such a case. Even if a node is blocked by one leader it may receive the StartColoring message by another leader and continue to request a color by this other leader. Another source of error in the dynamic setting is, as in CRRCOR, the request state, in which the nodes depend on the one leader they selected. However, less nodes are affected, as the nodes are allowed to leave the request state if they get blocked by another node. Finally, the algorithm may benefit from the smaller deployment area more than the other algorithms due to its increased transmission range, however, we expect this to be not significant. Overall, the algorithm achieves a solid performance in the mobile setting.

MWCOLOR performs even a little better than YUCOLOR, which is probably mainly due to the increased number of colors and thereby the lower probability for a conflict. As before, some nodes may get stuck in the request state, as dominated nodes select one leader and keep trying to contact this leader, however, resetting the nodes to their first color competition state leads to significantly less time required to re-color the nodes than in CRRCOR.

The best performance in the mobile setting is achieved by RAND4DCOLOR, which maintains a very high fraction of validly colored nodes throughout the execution. This is due to the fast runtime of the algorithm and the fact that whenever a conflict is detected the nodes simply try to resolve it immediately. Thus, a very high percentage of validly colored nodes is maintained. Even for a node speed of 1.0 less than 4% of the nodes have an invalid color. This performance can be achieved as no structure needs to be build or maintained and detected conflicts are treated immediately by selecting a new random color.

4.7.1 Asynchronous Wake-up

Let us now examine the robustness of the algorithm with respect to some nodes in the network waking-up later than large parts of the network. We consider how many of the 500 already colored nodes are disturbed by another 100 to 500 nodes waking up in the network and executing the algorithm. We use the random deployment on an area of $1000\text{m} \times 1000\text{m}$ as in most parts of this paper. We consider the algorithms RAND4DCOLOR, RAND4DRESPCOLOR and the correcting variants CRRCOR and MWCOR and measure the number of already colored nodes that detect a conflict. We do not consider YUCOR, as YUCOLOR does explicitly not support the setting of nodes waking up in late stages of the algorithm (which also resulted in worse results in preliminary experiments). The results of the experiment are shown in Table 8. Note that the basic algorithms COLORRED and MWCOLOR are expected to produce no or a lot less disturbance as they support late wake-up of nodes and the reliable communication ensures that the nodes are aware of colors selected by neighbors. However, we consider the performance of the variants we optimized towards achieving a high performance regarding the runtime and the number of conflicts, thus, already colored nodes may be disturbed as we do not have reliable local broadcasting.

For RAND4DCOLOR the results indicate that even if additional 500 nodes wake up, only 32 already colored nodes detect a conflict. If we reduce the number of nodes starting that late, the number of disturbed nodes decreases further to 10.9, which corresponds to about 2% of the pre-colored nodes. Thus, although atheoretical considerations in [2] indicate that late wake-up of few nodes may introduce many conflicts we do not observe this here. The RAND4DRESPCOLOR variant of the algorithm, which additionally requires the nodes to listen for DURATION time slots before transmitting color messages, achieves essentially optimal results with only 0.2 to 2.3 disturbed nodes on average. The remaining correcting variants perform a lot worse, with roughly between 40 and 180

happen without mobility, there is a small fraction of nodes not finishing the algorithm in Fig. 2a

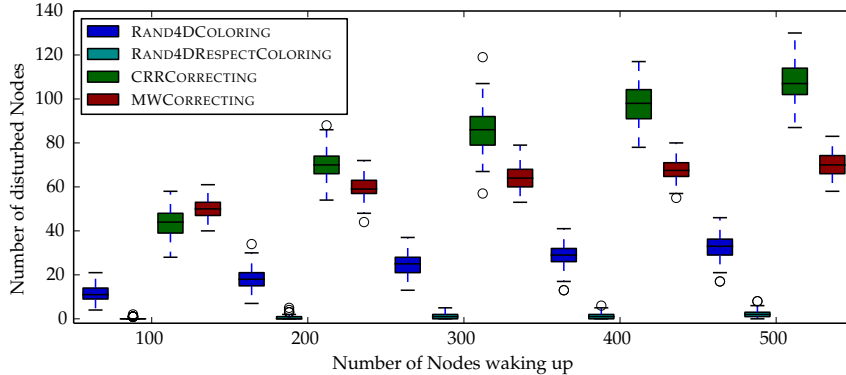


Figure 3: After 500 nodes finished and agreed on a valid coloring a varying number of additional nodes wake up (x-axis). We count the number of previously finished nodes that are disturbed, i.e. they detected a conflict after the additional nodes woke up.

Table 8: The average number of disturbed nodes for a varying number of nodes waking up after the 500 pre-deployed nodes selected a valid color.

Number of nodes waking up late	100	200	300	400	500
RAND4DCOLOR	11.24	18.05	24.97	28.51	32.39
RAND4DRESPCOLOR	0.14	0.72	1.12	1.54	2.34
CRRCCOR	43.79	69.84	85.89	98.05	107.54
MWCOR	50.12	59.89	64.55	67.93	70.15

disturbed nodes. However, we expect that adding an additional listening phase and preventing the nodes from selecting colors taken by neighbors would significantly decrease the number of disturbed nodes, similar to how RAND4DRESPCOLOR improved upon RAND4DCOLOR.

5 Conclusion

In this paper we experimentally evaluated several distributed node coloring algorithms designed for wireless ad-hoc and sensor networks. All algorithms operate under the realistic SINR model of interference and compute the valid colorings in $\mathcal{O}(\Delta \log n)$ time slots. We used the network simulator Sinalgo [1] to study the runtime and the number of conflicts in the computed colorings on several deployment scenarios. We conclude that our simple (4Δ) -coloring algorithm RAND4DCOLOR is very fast, requiring significantly less time than any other considered coloring algorithm. Our experiments additionally show that the algorithm is or can be made robust towards dynamic networks. Regarding $(\Delta + 1)$ -colorings, both COLORRED and RAND1DCOLOR are faster than the competing YUCOLOR algorithm. Our correcting variants improve the runtime for all algorithms, while preserving the relative ordering.

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A note on printing this Appendix: Printing Fig. 4 and Figs. 6 to 9 is relatively time consuming due to the complexity of the illustrations, however, it should still be printable in less than a minute per affected page.

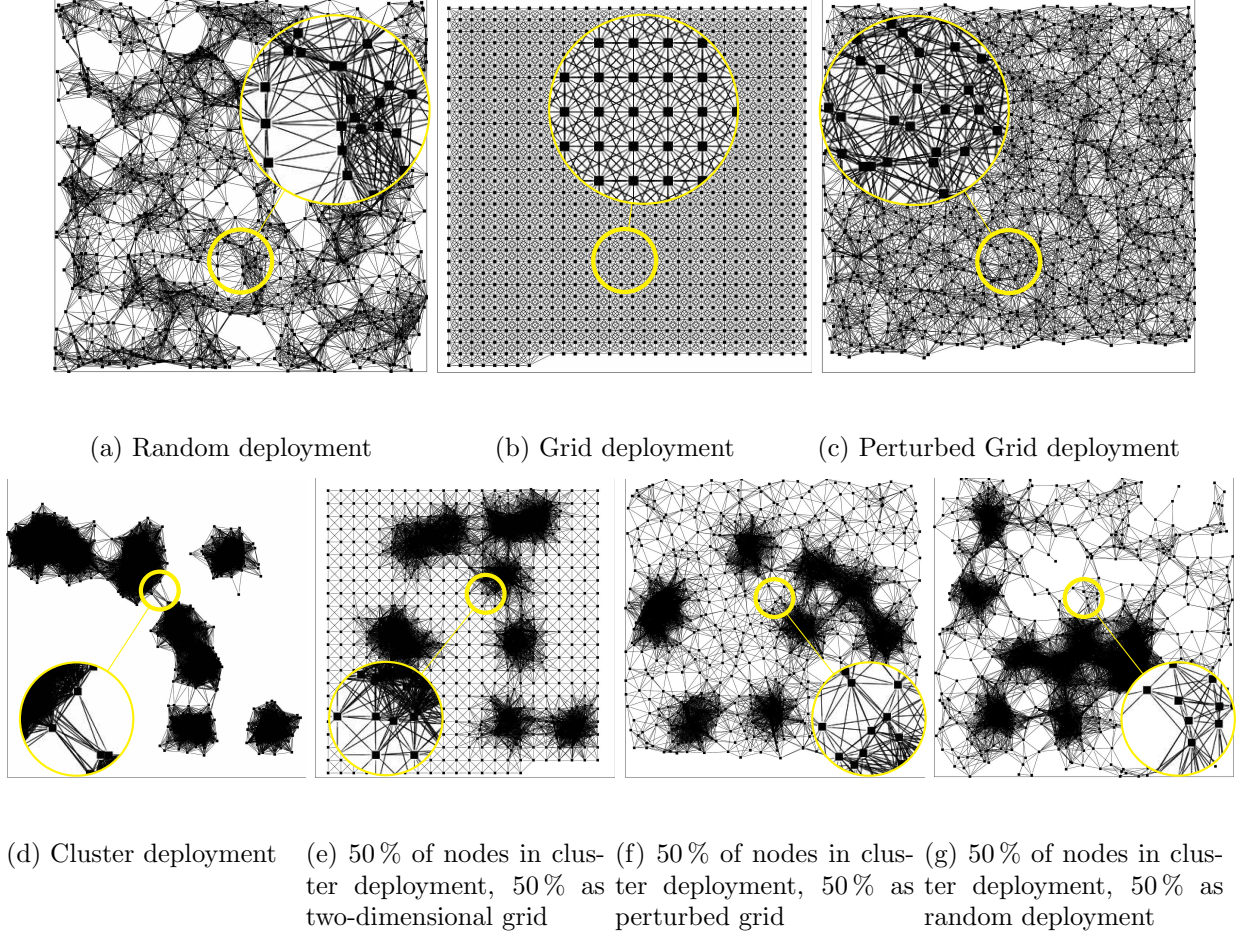


Figure 4: Sample networks illustrating our deployment strategies.

A Illustration of the Deployments

Visualizations of the deployments are depicted in Fig. 4.

B RAND4DCOLOR and the Length of Phases

Let us study the parameter `FACTOR` using the random deployment in our first experiment. We use values ranging from 0.001 to 1, corresponding to phase-lengths between 5 and 4600 time slots. Our results are depicted in Fig. 5.

We observe that both the runtime and the number of color redraws increases with the phase-length. Especially for the runtime this was expected, as our theoretical analysis guarantees the runtime of $\mathcal{O}(\Delta \log n)$ time slots only for a phase-length of $\mathcal{O}(\Delta)$, while `FACTOR` = 1 sets the phase-length to one round of local broadcasting, which is asymptotically in $\mathcal{O}(\Delta \log n)$. However, there is also a less formal intuition justifying the decreasing runtime for the decreasing phase-length. Once a node has detected a conflict (by receiving a message from a neighbor), it is not beneficial if the node must wait for the end of the phase before changing its color. Assume the node waits until the phase ends. It may happen that the node transmits its color to its neighbors, which may become aware of a conflict. If this happens, the respective neighbors also reset their color at the

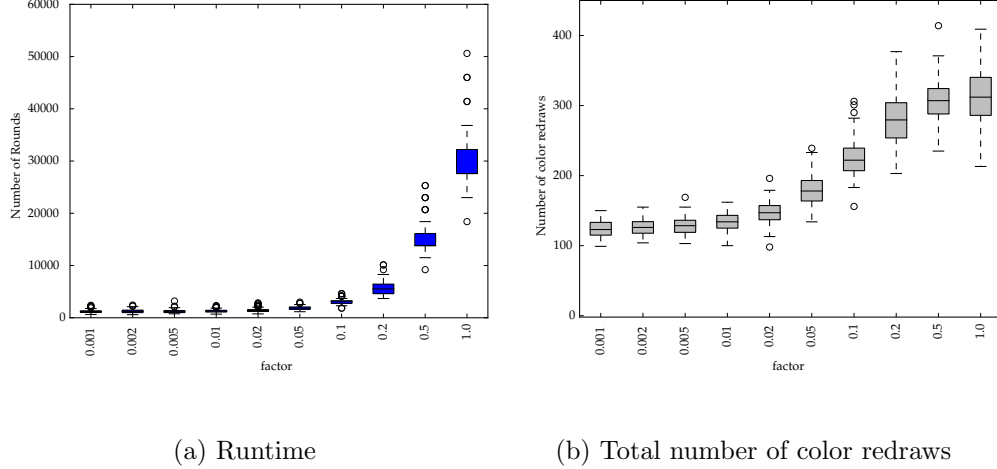


Figure 5: Determining the parameter FACTOR for RANDCDELTACOLORING, which influences the length of each phase. Both the runtime (left) and the number of color redraws (right) increase with increasing FACTOR.

end of their phase, although this conflict would be resolved without the neighbors intervention with significant probability at the end of the phase. On the other hand, dealing with the conflict directly does not introduce any penalty, as it is already determined that the detecting node resets its color. Hence, the shorter the phases are, the lower the runtime of the coloring algorithm.

Regarding the number of color redraws, we can observe something interesting. The longer the phases are, the more redraws are required, cf. Fig. 5b. This corresponds to the fact that the longer the phases are, the higher the probability that all conflicts are detected by the nodes. If a conflict is only detected by one of the conflicting nodes, the probability that it is resolved is already significant. Therefore, using phases of minimal length intuitively reduces the number of color redraws by a factor of two compared to phases of length DURATION. As each color redraw leads to some possibility of selecting the color of a neighbor we observe a factor of even slightly more than two in the total number of color redraws for long phases. Note that even longer phases do not lead to a further increase, as almost all conflicts are detected after phase-lengths that correspond to FACTOR = 1.

C Illustration of the Algorithms

In this section we illustrate the execution of RAND4DCOLOR, COLORRED, MWCOLOR, and YUCOLOR in Figs. 6 to 9 on the grid deployment to increase the readability. We use the parameters as described in Section 4 and refer to Appendix A for a more detailed visualization of a network using the grid deployment.

In the first section we describe the required modifications to Sinalgo to correct a flaw in the simulators SINR model implementation. In Appendix E we report results of experiments for the deployment strategies not presented in the paper

D Sinalgo - Patch for SINR Model

In this section we report on a modification of the SINR interference model that is delivered with Sinalgo version 0.75.3. The modifications are required to ensure that the SINR interference constraints are correctly evaluated. Without modification, the simulation framework considers for

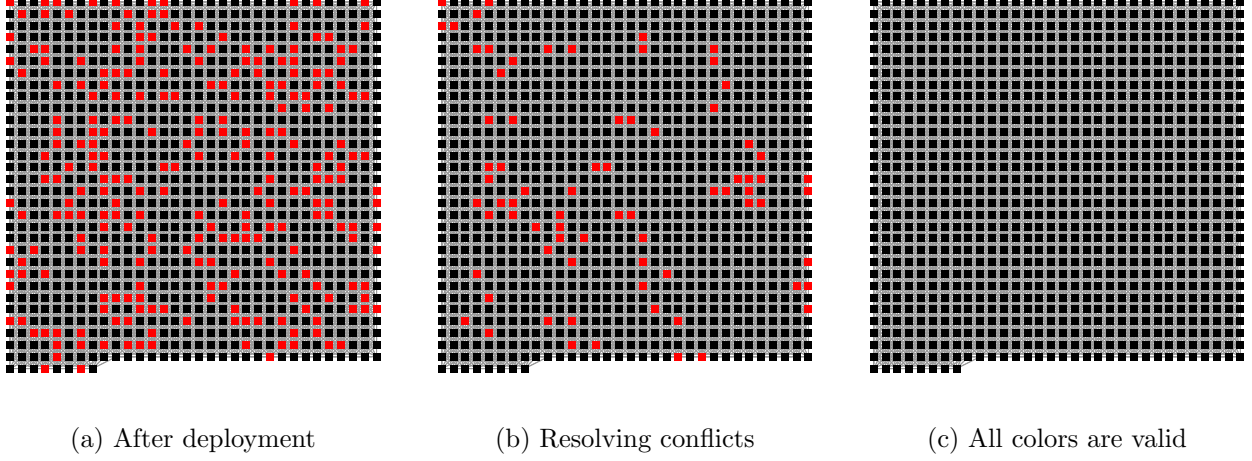


Figure 6: Illustration of an execution of RAND4DCOLOR. Black nodes have a valid color, red nodes are in conflict with one of their neighbors. We observe that even directly after deployment not too many conflicts exist (left). Furthermore, they get gradually eliminated (center) until all nodes have a valid color (right)

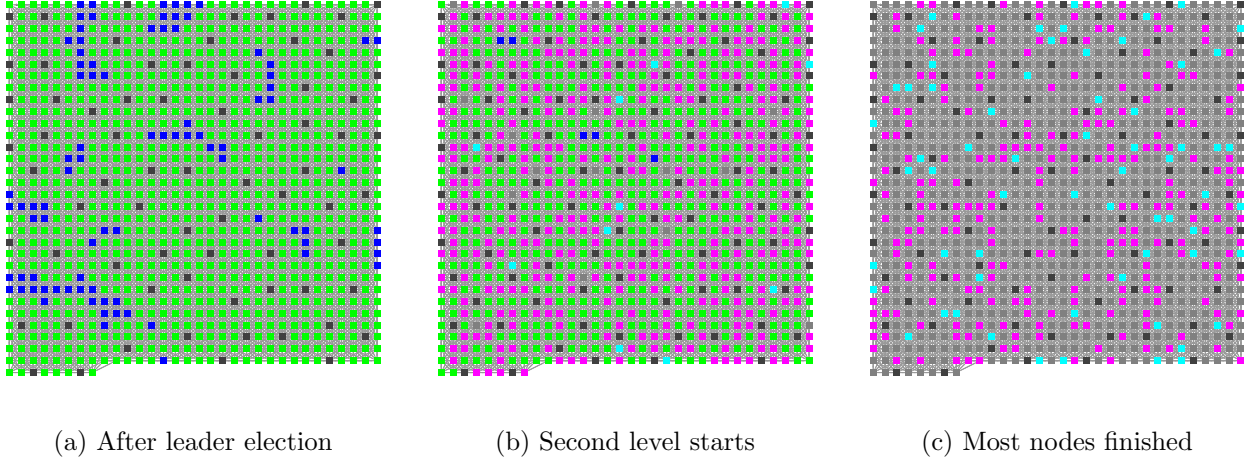


Figure 7: The flow of COLORRED. On the left most leaders (black) are computed. Green nodes are dominated and blue nodes still execute the first level MIS. In the center many dominated nodes received their active interval (magenta nodes), some cyan nodes compete in the second level MIS for a color. Very few gray nodes have already finalized their color by winning a second level MIS. On the right, most nodes selected their final color (gray), while others wait for their active interval or actively compete for selecting a final color.

a transmitted packet p the signal emitted during the transmission of the same packet p both as desired signal and interference. To correct this issue we equip each packet with a broadcast ID (or transmission ID), and ensure that we do only consider the interference of other packets (i.e., other transmissions). We state the modification in detail in Algorithms 2 to 4. The name of the algorithms gives the path to the respective file relative to the src folder of Sinalgo. The line number before and after the code marks the lines between which the code should be added (all line numbers are relative to the unchanged file).

Algorithm 2: projects/defaultProject/models/interferenceModels/SINR.java

```
116    /* detect if a packet is from the same broadcast as this packet.
    * If so, ignore the active packet.
    * If the broadcast id is -1, the packet is not from a broadcast,
    * and duplicate packets are found via pack == p
    */
    if (p.broadcastId != -1 && (pack.origin.ID == p.origin.ID &&
        && pack.broadcastId == p.broadcastId))
        continue;
    }
117
```

Algorithm 3: sinalgo/nodes/messages/Packet.java

```
104    /** broadcast id, allows to determine whether
    * 2 packets originated from the same broadcast
    */
    public int broadcastId;
105
194    pack.broadcastId = -1;
195
259    broadcastId = -1;
260
```

Algorithm 4: sinalgo/nodes/Node.java

```
77    import sinalgo.tools.Tools;
78
1487    int broadcastId = Tools.getRandomNumberGenerator().nextInt(100000);
1488
1495    sentP.broadcastId = broadcastId;
1496
```

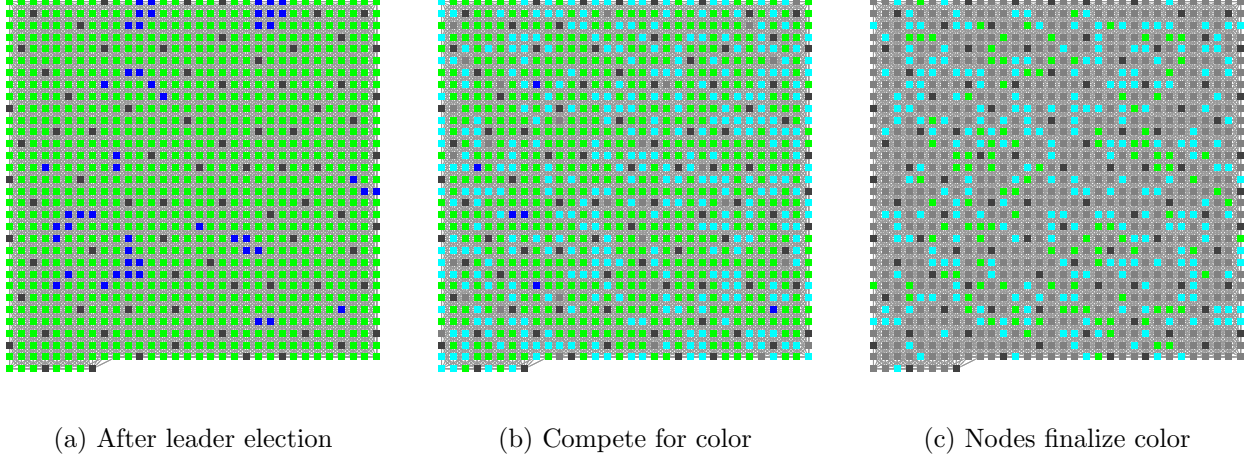


Figure 8: One execution of MWColor. On the left almost all leaders (black) are computed. Green nodes are dominated and request a color block, while blue nodes still compete in the MIS to become leaders. In the center many nodes received their color blocks to compete for a color (cyan) and some nodes already selected their final color (gray). On the right all leaders are computed and more nodes selected their final color.

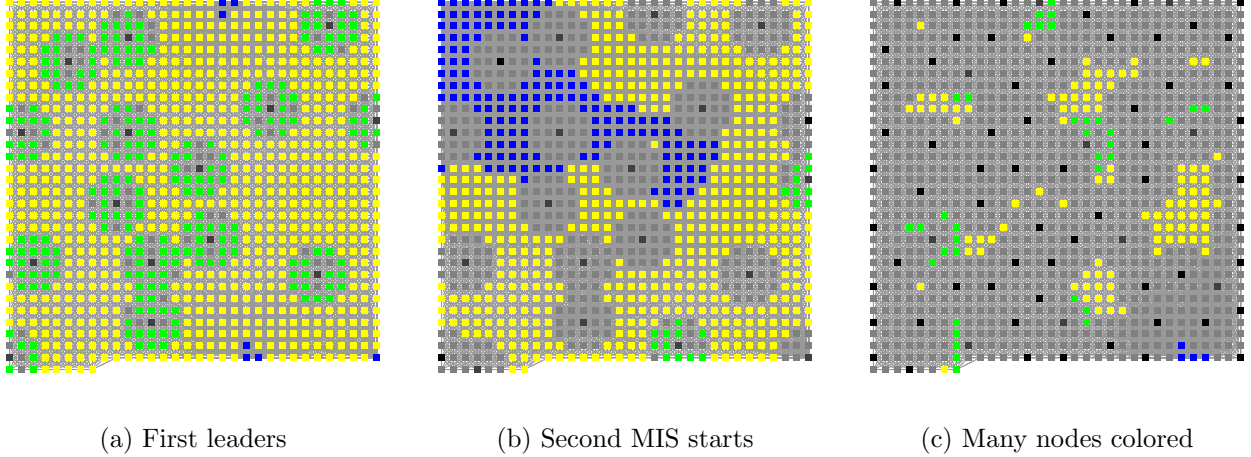


Figure 9: In the illustration of YUCOLOR the MIS is regarding a larger range. Thus, on the left only few leaders (black) dominate green nodes that request permission to select a color. Some nodes already selected their final color (gray). Most nodes are blocked (yellow), while some blue nodes still compete to enter the MIS. In the center, the first MIS nodes start to resign as leaders, allowing formerly blocked node to compete in the MIS again (blue nodes). On the right, only few nodes remain to be colored, some wait to receive the permission by their leader, most others are currently blocked.

E Experiments: Other Distributions

In the paper we showed detailed data only for the random deployment strategy, for other deployments we restricted ourselves to show only the overall best results. In this section we present additional data obtained from the experiments described in the referenced chapter. This data justifies our selection of the parameters as used to obtain the overall best results.

Let us briefly describe the contents of the tables. In each table we report the average number of conflicts and the average runtime and mark the best or the selected combination as bold. In Table 9 we consider variants of our phase-based (4Δ)-coloring algorithm RAND4DCOLOR, in which nodes simply select a new random color at the end of a phase if a conflict was detected during the

phase. For COLORRED we need to determine the parameter FACTOR, the results for different values are given in Table 10. For all deployment strategies we selected the same FACTOR of 0.6 to be an optimal balance between number of conflicts and runtime. In Table 11 we consider COLORRED and its variant CRRANDCOLOR, which replaces the valid color of each node with a random color. We observe that, despite differences in the average runtime and the number of conflicts, the results are similar and as for 2Δ available colors good results are achieved for all deployments. For the algorithms MWCOLOR and YUCOLOR we show the results for various values of the parameter FACTOR in Tables 12 and 13, respectively. We select FACTOR = 0.2 as optimal value for both algorithms and all deployment strategies.

The results of our heuristic improvements of CRRANDCOLOR, MWCOLOR, and YUCOLOR for different values of DURATION' are given in Tables 14 and 15. In these heuristics we reduce the number of conflicts by allowing nodes to reset to certain points in the algorithms once a conflict is detected. This increases the runtime for the standard DURATION, however, using a decreased DURATION' we can decrease both the average runtime and the average number of conflicts. The values selected as optimal are marked bold and use a 1/16 or 1/8 fraction of DURATION as DURATION'.

Table 9: Runtime and number of conflicts for RAND4DCOLOR and its variants.

Deployment	Algorithm	Runtime	Conflicts
Cluster	RAND4DCOLOR	3321	0.00
	RAND4DRESPCOLOR	15 639	0.00
	RAND4DFINALCOLOR	16 110	0.00
	RAND1DCOLOR	11 822	0.00
Cluster&Grid	RAND4DCOLOR	2186	0.00
	RAND4DRESPCOLOR	10 172	0.00
	RAND4DFINALCOLOR	10 456	0.00
	RAND1DCOLOR	8211	0.00
Cluster&PGrid	RAND4DCOLOR	2016	0.00
	RAND4DRESPCOLOR	9847	0.00
	RAND4DFINALCOLOR	10 199	0.00
	RAND1DCOLOR	6627	0.00
Cluster&Random	RAND4DCOLOR	2316	0.00
	RAND4DRESPCOLOR	10 175	0.00
	RAND4DFINALCOLOR	10 349	0.00
	RAND1DCOLOR	8153	0.00
Grid	RAND4DCOLOR	974	0.00
	RAND4DRESPCOLOR	4244	0.00
	RAND4DFINALCOLOR	4354	0.00
	RAND1DCOLOR	3358	0.00
PGrid	RAND4DCOLOR	1372	0.00
	RAND4DRESPCOLOR	6114	0.00
	RAND4DFINALCOLOR	6283	0.00
	RAND1DCOLOR	4548	0.00
Random	RAND4DCOLOR	1256	0.00
	RAND4DRESPCOLOR	5668	0.00
	RAND4DFINALCOLOR	5865	0.00
	RAND1DCOLOR	4174	0.00

Table 10: Average number of conflicts and average runtime for COLORRED using different parameters FACTOR. We report the values for each deployment strategy.

FACTOR	0.05	0.1	0.2	0.3	0.4	0.6	0.8
Cluster							
conflicts	0.00	0.04	0.02	0.17	0.18	1.04	2.74
runtime	902 421	468 046	240 713	163 781	126 039	88094	69 837
Cluster&Grid							
conflicts	0.00	0.00	0.06	0.17	0.20	1.05	3.15
runtime	590 431	297 464	151 579	103 595	80 001	55760	44 176
Cluster&PGrid							
conflicts	0.00	0.04	0.02	0.18	0.63	3.32	9.55
runtime	583 200	294 533	150 570	103 126	78 824	55157	43 619
Cluster&Random							
conflicts	0.00	0.04	0.04	0.04	0.30	1.31	4.31
runtime	578 623	292 450	150 245	102 685	78 946	55005	43 716
Grid							
conflicts	0.00	0.08	0.12	0.11	0.08	0.30	1.17
runtime	272 122	137 497	70 855	48 354	37 137	25770	20 258
PGrid							
conflicts	0.02	0.02	0.00	0.04	0.06	0.14	0.52
runtime	375 157	190 144	97 660	66 541	51 139	35624	27 964
Random							
conflicts	0.00	0.04	0.10	0.00	0.12	0.51	2.47
runtime	339 013	171 099	87 924	59 995	46 266	32224	25 384

Table 11: Average runtime for COLORRED and CRRANDCOLOR for colorings of different sizes. The runtimes are almost identical although CRRANDCOLOR uses only a random color to replace the valid coloring used in COLORRED.

Number of colors		$\Delta + 1$	2Δ	3Δ	4Δ
Cluster					
COLORRED	conflicts	4.99	0.88	0.88	0.64
	runtime	51 476	67 624	88 101	108 721
CRRANDCOLOR	conflicts	13.19	1.05	0.76	0.67
	runtime	53 566	67881	88 529	109 178
Cluster&Grid					
COLORRED	conflicts	2.56	1.08	0.98	1.31
	runtime	30 861	42 682	55 536	69 465
CRRANDCOLOR	conflicts	5.67	1.03	0.63	0.95
	runtime	31 713	42692	55 599	68 964
Cluster&PGrid					
COLORRED	conflicts	6.00	3.45	3.32	3.49
	runtime	32 024	42 094	55 148	68 682
CRRANDCOLOR	conflicts	10.62	3.23	3.77	3.67
	runtime	32 913	42385	55 091	68 015
Cluster&Random					
COLORRED	conflicts	2.63	1.53	1.34	1.17
	runtime	30 337	42 466	54 884	68 399
CRRANDCOLOR	conflicts	5.94	1.83	1.12	1.24
	runtime	30 939	42034	55 113	68 619
Grid					
COLORRED	conflicts	9.22	0.42	0.42	0.20
	runtime	16 128	20 310	25 929	31 297
CRRANDCOLOR	conflicts	31.01	0.82	0.42	0.32
	runtime	17 555	20367	25 798	31 257
PGrid					
COLORRED	conflicts	1.54	0.18	0.10	0.04
	runtime	20 367	27 669	35 683	43 631
CRRANDCOLOR	conflicts	5.31	0.10	0.10	0.15
	runtime	20 996	27817	35 682	43 633
Random					
COLORRED	conflicts	3.29	0.86	0.55	0.61
	runtime	18 638	24 824	32 197	39 737
CRRANDCOLOR	conflicts	6.52	0.93	0.65	0.63
	runtime	19 766	24758	32 287	39 696

Table 12: Average number of conflicts and average runtime for MWCOLOR using different parameters FACTOR. We report the values for each deployment strategy.

FACTOR		0.05	0.1	0.2	0.3	0.4	0.6
Cluster	conflicts	0.14	0.24	0.56	1.22	2.03	3.66
	runtime	197 790	111 416	73670	67 658	65 895	64 854
Cluster&Grid	conflicts	0.02	0.20	0.34	0.66	1.66	3.19
	runtime	130 064	73 089	47041	42 956	41 848	40 854
Cluster&PGrid	conflicts	0.16	0.28	1.06	1.47	2.9	7.89
	runtime	129 632	70 384	46142	41 730	40 823	38 883
Cluster&Random	conflicts	0.18	0.16	0.44	0.84	1.33	3.78
	runtime	129 266	74 177	46363	41 599	39 944	39 601
Grid	conflicts	0.02	0.06	0.26	1.16	2.04	4.1
	runtime	76 474	41 798	25456	20 680	18 918	17 221
PGrid	conflicts	0.04	0.06	0.28	0.48	0.84	1.14
	runtime	95 826	53 807	32812	27 421	25 652	23 765
Random	conflicts	0.10	0.12	0.42	1.02	1.48	2.71
	runtime	81 195	44 700	27982	23 995	22 807	21 870

Table 13: Average number of conflicts and average runtime for YUCOLOR using different parameters FACTOR. We report the values for each deployment strategy. C=Cluster, G=Grid, PG=PGrid, R=Random

FACTOR		0.05	0.1	0.2	0.3	0.4	0.6
C	conflicts	0.42	0.57	0.9	1.55	3.84	13.61
	runtime	382 860	233 591	164839	145 760	135 567	137 233
C&G	conflicts	0.5	0.52	0.94	2.19	4.2	15.68
	runtime	298 736	189 883	141003	127 842	127 151	120 294
C&PG	conflicts	0.56	1.20	1.30	3.56	9.03	36.42
	runtime	286 326	175 088	126488	112 753	107 833	109 203
C&R	conflicts	0.48	0.58	0.88	2.52	6.05	23.38
	runtime	277 447	172 869	122267	110 815	104 874	106 367
Grid	conflicts	1.04	1.06	2.01	3.7	6.16	20.48
	runtime	379 283	195 925	113105	86 592	73 141	60 833
PGrid	conflicts	1.00	0.96	1.12	1.83	3.04	10.35
	runtime	402 421	214 349	129054	100 451	88 521	76 147
Random	conflicts	0.62	0.71	1.39	2.90	6.41	22.43
	runtime	286 167	160 088	99946	82 707	72 660	67 131

Table 14: Average number of conflicts and average runtime for the correcting variants CRRCOR, MWCOR, and YUCOR for different values of DURATION'. In this table: Deployments involving the cluster deployment

Fraction of DURATION		1/32	1/16	1/8	1/4	1/2	1
Resulting DURATION'		143	287	575	1150	2300	4600
Cluster							
CRRCOR	conflicts	0.00	0.00	0.00	0.00	0.00	0.00
	runtime	24 425	17 860	17802	25 023	40 185	68 493
MWCOR	conflicts	0.63	0.94	0.45	0.10	0.00	0.00
	runtime	30 931	21 637	23531	33 245	50 780	105 200
YUCOR	conflicts	3.13	0.74	0.14	0.02	0.00	0.00
	runtime	12 843	20849	31 481	49 630	92 327	176 551
Cluster&Grid							
CRRCOR	conflicts	0.00	0.00	0.00	0.00	0.02	0.00
	runtime	17 008	12 473	11333	15 642	24 069	43 139
MWCOR	conflicts	0.32	0.59	0.36	0.04	0.00	0.00
	runtime	20 963	14 189	13535	18 601	30 008	54 896
YUCOR	conflicts	3.13	1.23	0.22	0.02	0.02	0.00
	runtime	7615	15835	25 747	42 543	78 202	154 572
Cluster&PGrid							
CRRCOR	conflicts	0.00	0.02	0.02	0.00	0.00	0.00
	runtime	13 609	10896	11 133	15 440	23 863	42 658
MWCOR	conflicts	0.38	0.42	0.41	0.02	0.02	0.00
	runtime	19 031	13353	13 402	17 490	30 973	55 544
YUCOR	conflicts	3.11	0.64	0.17	0.02	0.00	0.00
	runtime	7681	14620	22 982	38 887	71 253	139 299
Cluster&Random							
CRRCOR	conflicts	0.00	0.00	0.00	0.00	0.00	0.00
	runtime	16 903	12 789	11884	16 009	24 272	42 773
MWCOR	conflicts	0.23	0.33	0.44	0.04	0.00	0.00
	runtime	22 256	14 631	12780	17 341	28 919	53 017
YUCOR	conflicts	3.74	0.81	0.20	0.08	0.00	0.00
	runtime	7391	15060	24 173	38 689	69 636	135 511

Table 15: Average number of conflicts and average runtime for the correcting variants CRRCoR, MWCoR, and YUCoR for different values of DURATION'. In this table: Grid, PGrid and Random deployment

Fraction of DURATION		1/32	1/16	1/8	1/4	1/2	1
Resulting DURATION'		143	287	575	1150	2300	4600
Grid							
CRRCoR	conflicts	0.00	0.00	0.00	0.00	0.00	0.00
	runtime	7538	5343	5113	7013	11 839	20 756
MWCoR	conflicts	0.04	0.12	0.21	0.02	0.02	0.00
	runtime	8300	5740	5741	8287	15 636	27 958
YUCoR	conflicts	5.86	1.77	0.26	0.09	0.02	0.04
	runtime	3625	8654	16 819	33 089	68 253	135 904
PGrid							
CRRCoR	conflicts	0.00	0.00	0.00	0.00	0.00	0.00
	runtime	10 804	7904	7017	9638	15 267	27 889
MWCoR	conflicts	0.08	0.12	0.12	0.06	0.00	0.00
	runtime	11 831	7925	7567	10 558	18 263	35 860
YUCoR	conflicts	7.09	2.15	0.63	0.14	0.07	0.02
	runtime	4770	11479	20 947	38 641	77 174	155 428
Random							
CRRCoR	conflicts	0.00	0.00	0.00	0.00	0.00	0.00
	runtime	8965	6984	6489	8883	14 348	25 218
MWCoR	conflicts	0.13	0.23	0.10	0.02	0.02	0.00
	runtime	11 065	7688	6834	9105	15 762	31 027
YUCoR	conflicts	4.62	1.23	0.28	0.09	0.00	0.00
	runtime	4370	9807	16 652	29 635	58 189	116 646